Of Tulips and Trumpets

Is Time Diversification a Myth or Reality? Does Time Horizon Affect the Tolerance for Risk?

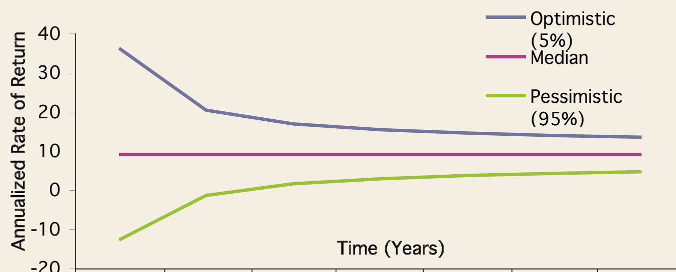
By Rex P. Macey, CIMA®, CFA®

The power of compounding returns over time and its effect on wealth is well-understood. With positive returns, wealth (cumulative return) increases with time. Does risk increase or decrease with time? The answer is fundamental to the questions of whether young investors can afford more risk and should investors become more conservative as they age. It's well-accepted that asset diversification reduces risk. Here I examine whether investing over time—allowing a mix of good and bad returns to smooth the result—provides a benefit known as "time diversification." I'll demonstrate that risk increases with time, making future levels of wealth more difficult to predict. I'll also unravel the contradiction that higher-risk portfolios may be less risky than lower-risk portfolios.

The illustration in figure 1 often is produced in asset allocation studies and is partly responsible for the misconception that risk decreases with time. Due to its shape, it sometimes is called a tulip chart. This tulip chart assumes a 10-percent annual expected return and a 15-percent expected risk (details of the assumptions are in the appendix).

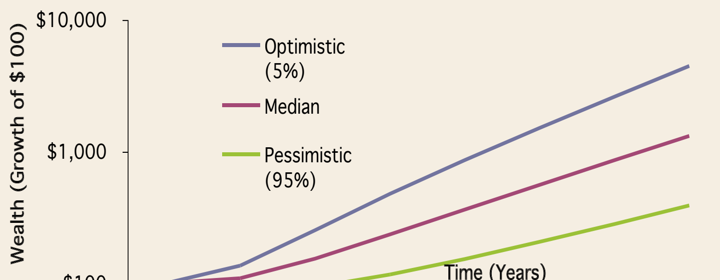
As the time horizon lengthens, the range between optimistic and pessimistic outcomes narrows. In this example, the 30-year optimistic and pessimistic values of 13.5 and 4.6 are closer than the one-year values of 36.3 and –12.8. If risk is defined as deviation from the average, then there appears to be less risk as time goes on. The tulip chart is numerically accurate but the risk reduction is illusory. The relationship between risk and time appears differently when one switches the focus from annualized return to cumulative return; that is, wealth.

FIGURE 1: TULIP CHART



	1	5	10	15	20	25	30
Optimistic (5%)	36.3	20.4	17.0	15.5	14.6	14.0	13.5
Median	9.0	9.0	9.0	9.0	9.0	9.0	9.0
Pessimistic (95%)	−12.8	−1.4	1.6	2.9	3.7	4.2	4.6

FIGURE 2: TRUMPET CHART



	0	1	5	10	15	20	25	30
Optimistic (5%)	$100	$136	$253	$479	$864	$1,519	$2,628	$4,496
Median	$100	$109	$154	$237	$364	$560	$861	$1,324
Pessimistic (95%)	$100	$87	$93	$117	$153	$206	$282	$390

Figure 2 is the trumpet chart. It presents exactly the same portfolio dynamics: a 10-percent return with a 15-percent standard deviation. Figure 2 illustrates the growth of $100 over time. If you subtract $100 from the values in the figure, you have the cumulative return in percent (e.g., the five-year median cumulative return is 54 percent). In contrast to figure 1, which suggests more certainty over time, figure 2 demonstrates that the longer your time horizon the less certain you can be about your wealth and cumulative return.

Wealth is what matters because investors spend wealth; they don't spend annualized rates of return. Thus the trumpet chart trumps the tulip chart. Risk increases over time. As time passes the investor in a risky portfolio has a greater chance of being further from the wealth predicted by a 10-percent growth rate. If we think of future wealth as a location on a trumpet chart, then it's harder to predict where we will be. The good news, as evidenced by the increasing values of the pessimistic line, is that we can be confident that we will be moving in the direction of greater wealth.

If risk is defined as standard deviation of return, which is the convention for asset allocation studies, we see that risk increases with time. We can be more precise and gain greater insight by noting that risk increases with the square root of time. As time quadruples, risk doubles. For example, if the standard deviation of returns is 10-percent per year, the standard deviation of returns over four years will be 20 percent, over nine years it will be 30 percent, and so forth. Risk doesn't grow linearly because of the tendency to experience a mix of good and bad results rather than a string of all good or all bad outcomes. The mix of good and bad tends to keep the result closer to the median.

But although risk increases with the square root of time, average return grows directly with time. In figure 2, it's clear that wealth grows log-linearly with time; that is, on a log scale it's a straight line.[1] For our discussion, it's only important to show that (cumulative) risk grows slower than (cumulative) return.

Since risk grows at a slower rate than cumulative return, the ratio of return to risk improves with time. Figure 3 illustrates an important investment implication of this.

FIGURE 3: PORTFOLIOS WITH HIGHER RISK CAN BE LESS RISKY

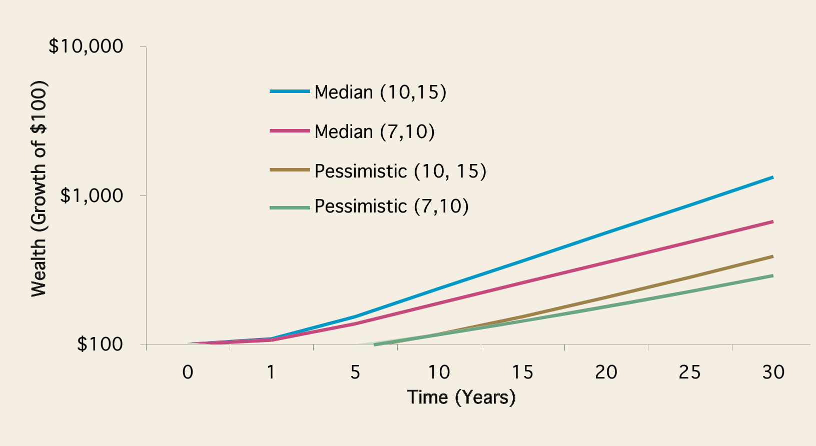


Figure 3 illustrates the seemingly contradictory statement that an investor with a long horizon may take less risk by taking more risk. It shows the median and pessimistic values for two different portfolios. One has the 10-percent return and 15-percent standard deviation (10, 15) that we've been using. The other has a lower return (7 percent) and a lower risk (10 percent). The key is that the pessimistic value of the more-risky portfolio exceeds that of the less-risky portfolio after about 10 years. If we change the definition of risk so that it means the probability of the portfolio value being below a specified value at some date in the future, the higher standard deviation portfolio may be less risky. This is why long-term investors may want to take more risk and why a retiree with a horizon spanning decades should continue to hold risky assets.

Let's compare the supposed benefits of asset diversification with time diversification. Imagine two investors each with $100 to invest over 20 years. The first invests in stocks for 10 years and then switches to bonds for 10 years. Over the 20-year period this investor averages 50 percent in stocks and 50 percent in bonds. The second investor maintains a 50-percent stock and 50-percent bond allocation over the entire 20 years. Assuming a less than 1.0 correlation between stocks and bonds will mean that the second investor can expect more wealth. The reduction in risk due to the imperfect correlation of returns causes the geometric return of the second investor's portfolio to be higher than the first.

Summary

Time diversification, if defined as the smoothing of returns over time, is illusory. Risk as defined by standard deviation is not reduced over time. Risk increases over time, just at a rate slower than return. Figure 2 shows that wealth is less certain over time. In many cases, investors may expect that a higher average expected return portfolio will have a higher value than a lower expected return, lower risk portfolio depending upon the assumptions and time horizon.

Appendix: About the Numbers

I make the same assumptions made in such models as the Black-Scholes options model: namely that returns are log-normally distributed and there is no serial correlation. In my opinion, these assumptions are approximately and generally true. They provide a useful model. The reality is somewhat different. In practice, extreme returns, high and low, occur more frequently than the log-normal model predicts. Serial correlation, which would mean that the past can be used to predict the

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future, generally is absent, but some would argue that there is mean reversion after extremes and sometimes there appears to be momentum. Despite the drawbacks, the assumptions made and the inaccuracies they create are sound enough for the purposes of the discussion of time diversification over long periods.

The astute observer will note that the median return is only 9 percent despite an assumed 10-percent return. The 10-percent assumption is an arithmetic average. That's the return expected in any given year. However, the effect of compounding will cause the realized (i.e., geometric) return to fall short of this. To appreciate this, consider the compound average return of +25 percent and –5 percent (i.e., one standard deviation above and below 10 percent). If you earn 25 percent and lose 5 percent, your wealth grows by 18.75 percent (0.1875 = [1.25 x 0.95]–1). This is below the value of 21 percent (0.21 = [1.1 x 1.1]–1) you would have achieved if you earned 10 percent in both years.

Also, note positive skewness in the distribution. The optimistic values (5 percent) are further from the median than the pessimistic values (95 percent). For example, at 25 years we have optimistic, median, and pessimistic returns of 14.0, 9.0, and 4.2 respectively. The optimistic value is 5-percent higher than the median value, while the pessimistic value is only 4.8-percent below the median value. This asymmetry is a property of the log-normal distribution and reflects the property that returns can exceed +100 percent but may not go below –100%.

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Endnote

1 See "Displaying the Growth of Assets Properly" in the inaugural Geek Speak column in *Investments & Wealth Monitor* (September/October 2008): 39. For the mathematically inclined w=(1+r)t so ln(w)=t*ln(1+r), which means that the log of w (wealth) increases proportionally with time.